Thermohaline circulation

Part 1
Main references

The Sea: Vol 10. The global coastal Ocean: Processes and methods

Chapter 7: Temperature distribution and the Seasonal thermocline Sections 7.1, 7.2 7.3

G.T Csanady: Circulation in the coastal ocean. Chapter 7. Thermohaline circulation Sections 7.0, 7.1
Some definitions

- **Buoyancy (reduced gravity)**
  \[ b = -g \frac{\rho - \rho_0}{\rho_0} = -g \varepsilon = g' \]
  
  \( \varepsilon \): fractional excess density with respect to a reference density
  
  \( b \): apparent weight of a water particle of density \( \rho \) surrounded by a medium having density \( \rho_0 \)

- **Baroclinic**
  state of a fluid in which isopycnals and isobars are mutually inclined

An horizontal buoyancy gradient (baroclinic density field) contributes to the horizontal pressure gradient:

\[
\frac{1}{\rho_0} \nabla_H p = \nabla_H \int_z^\eta \frac{g \rho}{\rho_0} d\zeta = g \nabla_H \eta - \int_z^0 \nabla_H b \, d\zeta
\]

Barotropic  Baroclinic
Buoyancy Flux $B$.
Buoyancy is input to the ocean at the surface or laterally through the heat or the fresh water flux.

$$ B = \frac{g \alpha_T}{\rho c_p} \left( Q_s - Q_b - Q_h - Q_e \right) + g \beta \left( P - E - R \right) S $$

Where

$$ \left( Q_s - Q_b - Q_h - Q_e \right) = Q $$  surface heat flux (see later for explanations)

$P =$ precipitation
$E =$ Evaporation
$R =$ river runoff

$\alpha_T =$ Thermal expansion coefficient

$\beta =$ Saline contraction coefficient
**Some definitions**

**Brunt-Vaisala Frequency.**
Water parcels displaced vertically are subject to a buoyancy restoring force that cause them to oscillate around the neutral stability level with a Frequency N:

\[
N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} = \frac{\partial b}{\partial z}
\]

**Richardson number**
Vertical mixing can occur in presence of a vertical buoyancy gradient only if sufficient Kinetic energy is produced to overcome the buoyancy restoring force. A measure of this requirement is the Richardson number:

\[
Ri = \frac{\partial b/\partial z}{(\partial u/\partial z)^2}
\]

Turbulent mixing in general occurs for \( Ri < 0.25 \)
The temperature distribution of any region of the ocean is controlled by:

- Net Heat flux trough sea surface
- Heat exchange by advection and diffusion with adjacent regions.

Heat flux gain: solar radiation (received directly or reflected/scattered from clouds/atmosphere)
Heat flux losses longwave back radiation, latent and sensible heat flux

Advevtive changes (positive and negative) of heat content determined by currents (horizontal and vertical)

Diffusive heat flux due to horizontal/vertical temperature gradients.

Vertical diffusive flux from small scale turbulent mixing or, if surface layer cooled, convective overturning

Horizontal diffusive heat exchanges from eddy motions at various space scales
Heat budget of oceanic and coastal areas

\[ Q_T = Q_s - Q_b - Q_h - Q_e - Q_v \]

let also:

\[ Q = Q_s - Q_b - Q_h - Q_e \]

**\( Q_s \):** Heat flux from incoming solar radiation.

**\( Q_b \):** Heat flux lost by longwave (back) radiation

**\( Q_h \):** Sensible heat flux (conduction)

**\( Q_e \):** Latent heat flux associated to evaporation
Temperature response to Heat budget variation

Open ocean response.

Let $\delta q$ be the quantity of heat absorbed by a unit volume of sea water.

The corresponding temperature increase $\delta T$ is given by:

$$\delta T = \frac{\delta q}{\rho c}$$

$c$: specific heat (constant pressure)

If the heat gain occurs in a time $\delta t$, then the temporal rate of temperature increase is

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c} \frac{\partial q}{\partial t}$$

The equation:

$$Q_T = Q_s - Q_b - Q_h - Q_e - Q_v$$

Denote the rate of gain of heat of a vertical seawater column (unit surface area). If its height is $h$, then:

$$\frac{\partial q}{\partial t} = \frac{Q_T}{h} \quad \text{and} \quad \frac{\partial T}{\partial t} = \frac{1}{\rho c \cdot h} \frac{Q_T}{h}$$
Temperature response to Heat budget variation

Open ocean response.

However, a positive net heat gain through the surface ($Q$) is absorbed by a surface layer with a thickness of only a few metres, therefore for a positive $Q$ only a thin surface layer become warmer and less dense than the water below.
Wave motion and wind stress generate turbulent energy to mix the heat input downward through a well mixed layer to a depth extending several tens of metres.
Below such depth turbulence cannot mix the heat further down and a (seasonal) thermocline is formed at the base of the mixed layer.
Open ocean response.

At temperate and polar latitudes the seasonal variation of $Q$ determine alternating periods of positive and negative heat flux, involving the formation (summer) and destruction of the seasonal thermocline.
At tropical latitudes $Q$ is permanently positive and therefore the “seasonal” thermocline is a permanent feature.

Almost universal Occurrence in the mid-latitude open ocean
Coastal ocean response.
The process leading to the formation/destruction of the seasonal thermocline are acting also on the coastal ocean, but turbulence generated by the bottom friction (particularly in areas affected by significant tidal dynamics) may provide a sufficient level of energy to keep the water column always in well mixed conditions, and the development of the thermocline depend mostly on the bottom depth and on the levels of mixing.

It has to be considered that shallower water column are heated/cooled up fastly than deeper and the interplay between mixing and heating can generate very different conditions.

Temperature response to Heat budget variation
Irish sea

Temperature response to Heat budget variations

Larger cooling
In coastal areas

WINTER

SUMMER

Larger warming
In coastal areas
Areas with reduced mixing:
Stronger thermocline in coastal areas with respect to the off-shore

Temperature section
Areas with strong mixing.
Coastal zone (under cooling processes) is kept well mixed while the offshore develop stratification.
Stratification in the coastal ocean

\[ B = \text{surface Buoyancy flux (input) through heat (warming) or freshwater (precipitation)} \]

Buoyancy input prevail over mixing. Stratification develops.

\[ B = \text{surface Buoyancy flux (extraction) through heat (cooling) or freshwater (precipitation)} \]

Buoyancy extraction prevail. Dense water are formed.
Horizontal patterns of temperature distributions (as well as salinity’s…seen later) originates Density (and hence pressure) horizontal gradients, giving rise to a dynamic that, since it depends on heat and fresh water fluxes is defined as “Thermohaline”,

The differential heating/cooling of the coastal water with respect to the offshore generate a well defined structure called “front”, mostly characterised by a sharp horizontal density difference, Determined by temperature and/or salinity, but also marked by other characteristics such as:

- Change of seawater color due to phytoplankton and detritus accumulation on one side of the front
- Foam accumulation near the front
Fronts

Temperature and Phytoplankton gradients

Color changes marked by A foam line

![Temperature and Phytoplankton gradients](image1)

![Color changes marked by A foam line](image2)
Front features are determined by the currents patterns in its vicinity.
Consider the vertical structure shown below (having depth $H$)

Resembling in an idealised way the structure previously seen.

An idealised S-shaped interface marking the contact between water with different density.
This structure

Originating from an initial situation determined by a membrane inserted into the fluid at \( x=0 \).

\[ Q_1 < Q_2 \]
Density in the shore side region \((-l\leq x\leq 0)\) is lower than the density in the offshore \((x\geq 0)\) region due to stronger heating and/or dilution processes.

The fractional density difference \(\varepsilon\) is given by:

\[
\varepsilon = \frac{\rho_2 - \rho_1}{\rho_2}
\]

The fluid depth in the “shore” region therefore increase from \(H\) to \(H(1+\varepsilon)\).

The lighter (heated and/or diluted) stands higher on one side of the membrane and therefore has an excess potential energy per unit mass.

\[
\Delta E_p = \frac{1}{2} g\varepsilon H = \frac{1}{2} g' H
\]
Assume that (at $t=0$) the “membrane” is removed allowing water to move freely (but without mixing and friction) between the “heavy” and “light” columns.

Upon membrane removal motion starts with lighter water moving at “surface” upon denser water and with denser water moving at “depth” below the lighter water, thereby originating the S-shaped interface among different density water.

The motion velocity $c$ can be computed assuming conversion from Potential to kinetic Energy:

$$c = \left(\varepsilon gH\right)^{1/2} = \left(g'H\right)^{1/2}$$
Let references axes be taken as in the figure below (origin at the front, x-axis perpendicular to the front, y-axis along it, and z-axis vertically pointing upward.

$\rho_1, \rho_2$: density in the upper and lower layer (respectively.)

$\eta_1$: free surface elevation

$\eta_2$: elevation ($\leq 0$) so that $\eta_1 - \eta_2 = h_1$, the thickness of the upper layer

$u_1, u_2$: cross frontal velocities in the upper and lower layer (respectively)

$H = h_1 + h_2$

Coriolis and advective terms are disregarded....

Eddy viscosity is assumed constant
And exchanges between layers is assumed non-existential.
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Eddy viscosity is assumed constant
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Is assumed non-existent.

Under such assumptions, the equation of motion in the cross frontal direction reduces to:

\begin{align*}
0 &= -\frac{1}{\rho_1} \frac{\partial p_1}{\partial x} + K_z \frac{\partial^2 u_1}{\partial z^2} \\
0 &= -\frac{1}{\rho_2} \frac{\partial p_2}{\partial x} + K_z \frac{\partial^2 u_2}{\partial z^2}
\end{align*}

\begin{align*}
\frac{\partial p_1}{\partial x} &= \rho_1 K_z \frac{\partial^2 u_1}{\partial z^2} \\
\frac{\partial p_2}{\partial x} &= \rho_2 K_z \frac{\partial^2 u_2}{\partial z^2}
\end{align*}

Where \( K_z = A_v/Q_0 \)
Where pressures $p_1$ and $p_2$ are given by the hydrostatic equation:

$$\frac{\partial p_1}{\partial x} = g \rho_1 [\eta_1 - z]$$

$$\frac{\partial p_2}{\partial x} = g \rho_1 [\eta_1 - \eta_2] + g \rho_2 [\eta_2 - z]$$

Coriolis and advective terms are disregarded....
Eddy viscosity is assumed constant
And exchanges between layers is assumed non-existent.
Continuity equation for the two layers yields no net integrated flow in the upper layer, constant flow in the lower layer.

If the front is advancing with velocity $c$ (defined previously) over water initially at rest then:

$$q = -cH$$

Coriolis and advective terms are disregarded....
Eddy viscosity is assumed constant
And exchanges between layers
Is assumed non-existent.

N.B: The further assumption
$$\eta_1 << \eta_2$$
Is made, so that the limit of integration $\eta_1$ is taken as 0, and $\eta_1$ as $h_1$.

$$0 = \int_{-h_1}^{0} u_1 \, dz$$
$$q = \int_{-H}^{-h_1} u_2 \, dz = -cH$$
Coriolis and advective terms are disregarded....
Eddy viscosity is assumed constant
And exchanges between layers is assumed non-existent.

The further assumption are:

\[
\frac{\partial p_1}{\partial x} = \rho_1 K z \frac{\partial^2 u_1}{\partial z^2}
\]
\[
\frac{\partial p_2}{\partial x} = \rho_2 K z \frac{\partial^2 u_2}{\partial z^2}
\]

\[
h_1 << H \quad \frac{\partial u_2}{\partial z} = 0
\]

Velocity in the lower layer vertically uniform.

The stress at the interface is related to velocity in the lower layer through a quadratic law with a drag coefficient \( c_d \).

Therefore the boundary conditions are as follows:
Coriolis and advective terms are disregarded. Eddy viscosity is assumed constant and exchanges between layers is assumed non-existent.

Therefore the boundary conditions are as follows:

\[ \frac{\partial p_1}{\partial x} = \rho_1 K_z \frac{\partial^2 u_1}{\partial z^2} \]
\[ \frac{\partial p_2}{\partial x} = \rho_2 K_z \frac{\partial^2 u_2}{\partial z^2} \]

\[ \left. \frac{\partial u_1}{\partial z} \right|_{z=0} = 0 \]

\[ u \bigg|_{z=-h_1} = u_2 \]

or also

\[ A_v \left. \frac{\partial u_1}{\partial z} \right|_{z=-h_1} = c_d u_2^2 \]
Coriolis and advective terms are disregarded.
Eddy viscosity is assumed constant
And exchanges between layers
Is assumed non-existent.

Solution for the system are:

\[ u_1 = u_s \left( 1 - \frac{3z^2}{h_1^2} \right) \]

\[ u_2 = -2u_s \]

velocity at the interface is opposite to the surface velocity \((z=0)\)
and twice in magnitude.

The form of the interface \(\eta_2(x)\) is described by

\[ x = \frac{c_d \varepsilon \eta_2^4}{36K_z} \]
Convergence (accumulation of detritus)

Velocity profiles at various distances from the front