Tides in the coastal domain

Low tide

High tide

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Main references

K. F. Bowden. 
Physical Oceanography of coastal waters: 
Chapter 2: Tides and tidal currents 
Section 2.4 
Chapter 7: Temperature distribution and the 
Seasonal thermocline 
Sections 7.3
Effect of reduced depth on tides

A tidal wave travelling over a progressively reduced depth undergoes changes, whose magnitude may be estimated considering the rate of transport of energy. As seen previously the rate of energy transmission across an area $\delta x \delta z$ is given by:

$$p' u \delta x \delta z$$

with (for a long wave) $p' = g \rho \eta$ (excess hydrostatic pressure).

Assuming

$$\eta = A \cos(\kappa x - \sigma t) \quad u = U \cos(\kappa x - \sigma t)$$

and then integrating vertically the energy flow across a unit width section:

$$E = \frac{1}{2} g \rho H A U \cos^2(\kappa x - \sigma t)$$

that, recalling

$$U = \frac{c}{H} A = \left(\frac{g}{H}\right)^{1/2} A$$

Can be written as:

$$E = \frac{1}{2} g^{3/2} \rho H^{1/2} A^2$$

See lecture about Kelvin wave.
Effect of reduced depth on tides

\[ E = \frac{1}{2} g^{3/2} \rho H^{1/2} A^2 \quad \text{or also:} \quad E = \frac{1}{2} g^{1/2} \rho H^{3/2} U^2 \]

Consider now a tidal wave travelling from a basin having depth \( H_1 \) to a basin having depth \( H_2 \). The propagation velocity will be reduced according to:

\[ \frac{c_2}{c_1} = \left( \frac{H_2}{H_1} \right)^{1/2} \]

It is possible to get a rough estimate of the changes in elevation and velocity amplitude by assuming that the rate of energy transmission is conserved. Then:

\[ \frac{1}{2} g^{3/2} \rho H_2^{1/2} A_2^2 = \frac{1}{2} g^{3/2} \rho H_1^{1/2} A_1^2 \quad \Rightarrow \quad \frac{A_2}{A_1} = \left( \frac{H_1}{H_2} \right)^{1/2} \]

\[ \frac{1}{2} g^{1/2} \rho H_2^{3/2} U_2^2 = \frac{1}{2} g^{1/2} \rho H_1^{3/2} U_1^2 \quad \Rightarrow \quad \frac{U_2}{U_1} = \left( \frac{H_1}{H_2} \right)^4 \]
Effect of reduced depth on tides

\[
\frac{A_2}{A_1} = \left( \frac{H_1}{H_2} \right)^{1/2} \quad \frac{U_2}{U_1} = \left( \frac{H_1}{H_2} \right)^4
\]

Let \( H_1 = 4000 \) m, \( H_2 = 100 \) m then

\[
c_1 = 198 \text{ m s}^{-1}, \quad c_2 = 31.3 \text{ m s}^{-1}
\]

\[
A_2 = 2.51 A_1 \quad U_2 = 15.9 U_1
\]

Assuming

\[
A_1 = 0.5 \text{ m} \quad \text{and} \quad U_1 = 2.5 \times 10^{-2} \text{ m s}^{-1} \quad \text{then}
\]

\[
A_2 = 1.26 \text{ m} \quad \text{and} \quad U_2 \approx 0.40 \text{ m s}^{-1}
\]

Velocity is increased to a much greater extent than the elevation.
Effect of reduced depth on tides

CAVEAT!!!!!

The method described is a ROUGH estimate of the changes occurring when the tidal wave moves from the pelagic to the coastal domain because:

- Changes occurring on a sloping bottom cannot be inferred from an equation assuming constant depth.
- Part of the energy associated to the tidal wave will be reflected back by the sloping bed, so that less energy will be transmitted to the shelf.
- Part of the energy will be dissipated by bottom friction becoming more and more important as the depth decreases.
Effect of reduced depth on tides

In fact.....:

Friction and reflection reduce velocity.

Depth change 1000-100 m in ~10 km

Elevation and velocity increase across the Bottom "jump"
The vertical structure of the water column results from the competition between the buoyancy inputs (see specific lecture) and the stirring by the tides and wind stress. In areas dominated by the tidal forcing the criterion for the water column stratification development is as follows:

Let \( Q \) be the net heat flux per unit area of surface per unit time. The mass of water in a volume of unit horizontal area and thickness \( H \) is \( \rho h \).

Then the absorption of a quantity of heat \( Q \delta t \) in time \( \delta t \) will determine a rise in temperature:
The increase of temperature generates an increase of volume: given the unit area considered it is an increase of thickness $\delta h$. Then:

$$\delta h = \alpha h \delta T$$

corresponding to a density variation:

$$\delta \rho = \rho \frac{\delta h}{h} = \rho \alpha \delta T = \frac{\alpha Q \delta t}{c_p h}$$

If such layer (thickness $h$ and density $\rho_{-\delta \rho}$) is mixed with the lower layer (thickness $H-h$ and density $\rho_{Q}$), the variation in gravitational potential energy is given by:

$$\delta V = \frac{1}{2} g \delta \rho h (H-h) = \frac{1}{2} \frac{g \alpha Q \delta t}{c_p} (H-h)$$

And the rate of change in potential energy:

$$\frac{dV}{dt} = \frac{1}{2} \frac{g \alpha Q (H-h)}{c_p}$$
Tides and stratification
The heating stirring systems

\[
\frac{dV}{dt} = \frac{1}{2} g \alpha Q (H - h) \frac{c_p}{c_d}
\]

The rate of energy loss due to bottom friction is:

\[
\frac{dE}{dt} = \tau B U_B \quad \text{with} \quad \tau_B = c_d \rho |U_B| U_B
\]

then

\[
\frac{dE}{dt} = \tau_B |U_B|^3
\]

Assuming an harmonic constituent of the tide given by:

\[
U_B = U \cos \sigma t
\]

then averaging over a tidal period it follows:

\[
\frac{d\bar{E}}{dt} = \frac{4 c_d}{3 \pi} \rho |U|^3
\]

Assuming that the energy lost from the tidal current is is converted into turbulent kinetic energy and that only a fraction \( \varepsilon \) of this energy is available to increase the potential energy of the water column the condition for mixing occur if:

\[
\frac{dV}{dt} \leq \varepsilon \frac{d\bar{E}}{dt}
\]
Tides and stratification
The heating stirring systems

\[
\frac{dV}{dt} \leq \varepsilon \frac{dE}{dt} \quad \text{or} \quad \frac{dV}{dt} = \frac{1}{2} \rho c_p g \alpha Q (H - h) \frac{dE}{dt}
\]

To a first approximation, \( c_d, Q, c_p, g, \alpha \) and \( \varepsilon \) can be considered constant. Then the R.H.S of the above expression is constant.
Assuming further that \( h \ll H \) and that \( Q \) can be considered as well constant when comparing nearby areas during the same time period. Then the criterion for Complete vertical mixing under tidal action is:

\[
P \equiv \frac{H}{U^3} \leq P_{\text{crit}}
\]

With

\[
P_{\text{crit}} = \frac{8}{3\pi} \varepsilon \frac{c_d \rho c_p g \alpha Q}{H - h}
\]
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The above can be further generalised by expressing the time rate of change of potential energy in terms of buoyancy flux, allowing for a flux of fresh water through precipitation \((P)\), river runoff \((R)\) and evaporation \((E)\):

\[
B = g \left( \frac{\alpha Q}{\rho c_p} - \beta (E - P - R) S \bigg|_{z=\eta} \right)
\]

Where \(\beta\) is the coefficient of haline contraction and \(S \bigg|_{z=\eta}\) is the surface salinity value.

When a fresh water quantity \(-(E-P-R)\delta t\) is added to the surface layer of thickness \(h\), salinity decrease by a quantity:

\[
\delta S = (E - P - R) \delta t S \bigg|_{z=\eta}
\]

And the density by a quantity \(\delta \rho = \beta \delta S\).

The temporal rate of change of potential energy is then modified as follows:

\[
\frac{dV}{dt} = \frac{1}{2} \frac{g\alpha Q (H - h)}{c_p} \quad \text{becomes} \quad \frac{dV}{dt} = \frac{1}{2} \frac{B (H - h)}{c_p}
\]
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\[ \frac{dV}{dt} = \frac{1}{2} B (H - h) c_p \]

And the mixing criterion retaining the thickness of the surface layer is:

\[ P \equiv \frac{H - h}{U^3} \leq P_{\text{crit}} \]

With

\[ P_{\text{crit}} = \frac{8}{3\pi} \frac{c_d \varepsilon}{B} \]

\[ \delta S = \left( E - P - R \right) \delta t S \bigg|_{z=\eta} \]