Endogenous Cycles in a Stiglitz–Weiss Economy*

Javier Suarez

CEMFI, Casado del Alisal 5, 28014 Madrid, Spain

and

Oren Sussman

Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

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The literature on financial imperfections and business cycles has focused on propagation mechanisms. In this paper we model a pure reversion mechanism, such that the economy may converge to a two-period equilibrium cycle. This mechanism confirms that financial imperfections may have a dramatic amplification effect. Unlike in some related models, contracts are complete. Indexation is not assumed away. The welfare properties of a possible stabilizing policy are analyzed. The model itself is a dynamic extension of the well-known Stiglitz-Weiss model of lending under moral hazard. Although stylized the model still captures some important features of credit cycles. Journal of Economic Literature Classification Numbers: D82, E32, E51. © 1997 Academic Press

1. INTRODUCTION

Economic research in recent years has revitalized the idea that financial factors should play a central role in business-cycle theory.1 On the one hand, there exists a growing body of empirical work showing that financial imperfections affect real economic decisions2 in a way which varies systematically along the business-cycle. On the other hand, theoretical work—Bernanke

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1 Fisher [10] gives one of the first coherent statements; see King [19]. For many years, however, it was ignored: it is not quoted by Patinkin [22], the authoritative handbook of the 1950’s and 1960’s.

2 Usually, this literature shows that liquidity affects economic decisions, in contrast to the prediction of the Modigliani–Miller theorem where only net-present value matters. See Bernanke et al. [3] for an up-to-date survey.

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and Gertler [2] and more recently Kiyotaki and Moore [20]—shows how transitory shocks are propagated via imperfections in financial markets. The novel contribution of this paper is the modeling of an endogenous reversion mechanism, such that the economy may converge to a two-period equilibrium cycle. The model is kept deliberately simple so as to allow a transparent exposition of the mechanism. Indeed, the model is a dynamic extension of the well-known Stiglitz–Weiss [24] (henceforth SW) model of lending under moral hazard. 3

Let it be clear that we view the reversion mechanism as a complement to the propagation mechanism. Obviously, it takes both to produce a complete theory of business fluctuations. But further, we use our model to clarify some theoretical issues. First, it is often argued that financial imperfection provide a crucial amplification effect that can solve the “small shocks, large cycles” puzzle (see Bernanke et al. [3]). In our model the amplification effect is dramatic: the variance of external shocks is zero while output fluctuations may still be sizable.

Secondly, in both Bernanke and Gertler [2] and Kiyotaki and Moore [20] the external shock is not anticipated in advance. Hence, agents do not hedge in precaution. It seems, however, that agents who fail to foresee a repeated shock do not have rational expectations. That raises the concern that irrationality is an indispensable ingredient within such a theory. We show that this is not the case. In our model the whole sequence of future prices is rationally (and perfectly) foreseen. Moreover, contracts are “complete” and all relevant information (future prices included) is internalized. To the best of our knowledge our example is the first clear-cut demonstration of a cycle generated solely by financial imperfections, without any modification of the rationality assumption.

More to the point, the issue at hand is that of “indexation.” Consider, again, Kiyotaki and Moore [20]. 4 The external shock operates via a price decline that decreases the value of collateral and, hence, borrowing capacity. It is well known that in that case insurance can easily be provided by price indexation. 5 Since such indexation is mutually beneficial, assuming

3 SW also contains a, maybe better known, adverse-selection section.

4 The problem goes back to Fisher [10]. In his view, the Great Depression was a result of money-price deflation that increased the real value of corporate debt, drained capital out of the corporate sector, which caused an adverse supply effect. But the initial effect can be indexed away to the mutual benefit of lenders and borrowers. Note that Fisher's explanation has two ingredients: lack of indexation and financial imperfections (which create the link between corporate wealth and supply). We show that the second effect is sufficient for a business-cycle theory. Needless to say, the first ingredient is extremely problematic. It may have been one of the reasons that prevented a serious consideration of Fisher's theory for so long.

5 Indexation is assumed away for the shock period only. All subsequent price dynamics is indexed.
it away is hard to justify. In our model, the economy slumps due to endogenous reversion and that happens despite the fact that contracts are optimally designed on the basis of a perfect foresight of future prices. It follows that financial factors can affect business fluctuations even without assuming indexation away.

And finally, we use our model in order to clarify the role of a potential stabilizing policy: its existence and welfare evaluation. (Note that it is full rationality and completeness of contracts which open the way for the welfare analysis.) We start the analysis of this part by demonstrating that there exists a stabilizing policy, which can be interpreted as an ordinary demand policy. Then, we show that when costs and benefits are aggregated, the policy produces a net positive surplus. But, unfortunately, the surplus cannot be lump-sum redistributed so as to generate a Pareto-dominating allocation. The reason is that lump-sum transfers do not exist in our model: because rents and liquidity matter, any reallocation of wealth affects real economic decisions.

As noted, our model is, essentially, a dynamic extension of the SW model with overlapping two-period projects. Hence, external finance generates excessive risk taking (i.e., above first-best probability of failure). Entrepreneurs face a downward sloping demand schedule so that prices fall when quantities boom. So here our story follows: boom production leads to low prices, which generates low liquidity and increases external finance. That leads to excessive risk taking and a high rate of failure—a bust. When quantities decrease, prices increase, liquidity flows in and the moral-hazard problem is mitigated. Low levels of risk taking will expand the industry, and it all starts over again.

It is important to stress that our model is “clean” in the sense that it contains no unusual ingredient that drives the result. As mentioned above, contracts are fully endogenized. The asymmetric information structure is a simple textbook moral-hazard problem. The basic story about the relation between external finance and risk taking comes from SW. The extension of project duration to more than one period is just an ordinary (and realistic) feature of capital theory. Preferences are standard and display risk neutrality in the numeraire good. In addition, we take a precautionary measure in

6 We believe our model can be used in order to defend the Kiyotaki-Moore model against such criticism: had their economy slumped due to endogenous reversion, there would have been no need to assume indexation away.

7 After SW, the words risk and probability of failure are used interchangeably.

8 Some elements of this story can be found in Sussman [25].

9 There is some similarity with the cobweb model, but, with differences in two major respects: first, rationality of expectations, and second, the cobweb model is a partial equilibrium model. Nevertheless, the general equilibrium characteristics of our model are too primitive to be emphasized.
order to assure that our result is properly interpreted: we prove formally (in Appendix A), that output quickly converges to a stationary level once the moral hazard is removed. Hence, the cycle results from the financial imperfection.

We have already made clear that the primary goal of this paper is theoretical. Obviously, it does not produce realistic time series. Yet, it is not without empirical value. A salient feature of the business cycle is that it is usually accompanied by a “credit cycle:” profits tend to decline towards the peak of the cycle, and the “liquidity crunch” leads the economy into the bust.\(^{10}\) The essence of this story is captured in our model: it is the high quantities of the boom which depress prices and create the liquidity shortage that increases the propensity to default that ends in a bust.

It is also noteworthy that our model can, in principle, be calibrated. The main behavioral relationship—the inverse relation between liquidity and default risk—is observable and can be estimated. Indeed, Holtz-Eakin et al.\(^{17}\) examine the wealth effect of an “exogenous” windfall (bequest) on the probability of survival of individual entrepreneurs. They find a significant positive effect which is consistent with our modeling. Needless to say there is, still, much work to be done before the model is ripe for calibration.

There are two other branches of the literature which deserve to be mentioned. Boldrin and Woodford\(^{6}\) survey the general equilibrium theory on endogenous fluctuations \(a la\) Day\(^{8}\) or Grandmont\(^{12}\). They argue that although endogenous cycles are compatible with complete markets, some “friction” (financial or other) is probably needed in order to get empirically relevant results. A step in that direction is taken by Woodford\(^{26}\) (see also Bewley\(^{4}\) and Scheinkman and Weiss\(^{23}\)). In his model equilibrium dynamics may be chaotic, but financial structure is crude and exogenously determined. Secondly, there is a growing literature, mostly of a static nature, on more realistic features of financial structure and aggregate economic activity. Many emphasize the role of the banking system, and describe mechanisms by which aggregate economic activity may be affected by changes in the cost of financial intermediation or by the level of banks’ capitalization.\(^{11}\) Some authors have stressed the role of bankruptcy costs (Greenwald and Stiglitz\(^{14}\)). Others still have remarked on the role different financial instruments (i.e., debt and equity) play in the “transmission mechanism” (see \(^{14}\) or King\(^{19}\)).

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the contract problem and shows the relationship between liquidity and risk taking. Section 4 derives the aggregate supply and defines a market equilibrium. Section 5 discusses the existence and stability of equilibrium.

\(^{10}\) See Gertler and Gilchrist\(^{11}\), Kashyap\(^{et al.}\)\(^{18}\) and Bernanke\(^{et al.}\)\(^{3}\).

\(^{11}\) See Bernanke\(^{1}\), Bolton and Freixas\(^{7}\), Blum and Hellwig\(^{5}\), Holmstrom and Tirole\(^{16}\).
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2. THE MODEL

Consider an infinite horizon, discrete time \((t = 0, 1, \ldots)\) economy with two goods. One is a numeraire good which is used for both consumption and investment, the other is a perishable staple good which is used for consumption only. We call it coffee.\(^{12}\) At each date there is a perfectly competitive spot market in which coffee is exchanged for the numeraire good at a price \(p_t\).

There are two types of agents in our economy: entrepreneurs (who grow coffee) and consumers-lenders (who consume coffee and provide external finance to the entrepreneurs). Consumers are identical and live forever. They consume both the numeraire and coffee and they are risk-neutral in terms of the numeraire good:

\[
E_t \left\{ \sum_{s=0}^{\infty} \frac{1}{1+r} \left[ x_{t+s} + u(c_{t+s}) \right] \right\},
\]

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\(x_{t+s}\) and \(c_{t+s}\) are the consumption of the numeraire and coffee, respectively, at date \(t+s\); \(u(c)\) is an increasing and concave utility function; the constant \(r > 0\) is the rate of time preference; \(E_t\) denotes expectations formed at date \(t\). Let \(a_t\) denote the amount of (period \(t\)) external finance they supply; and let \(\bar{R}_t\) be the gross (random) rate of return per unit of finance extended at period \(t\) (to be determined by the contract problem below). Then, the consumers’ budget constraint is

\[
x_t + p_t c_t + a_t = e + \bar{R}_{t-1} a_{t-1}.
\]

Suppose that consumers’ endowments are such that the solution to their problem is always interior.\(^{13}\) Their behavior is characterized by two simple behavioral functions: (i) a time-invariant perfectly elastic supply of lending at the expected gross rate of return \(1+r\) (namely, \(r\) is the riskless rate), and (ii) a time-invariant downward sloping demand for coffee, \(D(p_t)\), which

\(^{12}\text{We use this name to hint that the coffee sector may be interpreted as a small open economy which is highly dependent on the production of a single staple good.}\)

\(^{13}\text{It is sufficient to assume that the endowment} e \text{ is large enough to cover the entrepreneurs’ financing requirements at each date.}\)
will only depend on the spot price of coffee at each period. The one-period indirect utility function of the consumers can be written as

\[ U(p_t), \]

where, by Roy’s Identity,

\[ U'(p_t) = -D(p_t). \]  \hspace{1cm} (3)

At each period a measure one continuum of entrepreneurs is born, each of which lives for three periods. They consume no coffee themselves, and have linear preferences in the numeraire good; their rate of time preference, \( r \), is the same as the consumers’. (Hence, given that \( r \) is the riskless rate, entrepreneurs are indifferent about the timing of consumption.) Entrepreneurs have exclusive access to the production technology of coffee: each is endowed with a single, indivisible, project that can be activated by investing one unit of the numeraire good. Once invested, this amount is sunk.

Once activated (at the entrepreneurs’ first period of life), a project has two production periods. In the first period it yields \( Y > 0 \) units of coffee deterministically. In the second period it yields \( Y \) units of coffee in case of “success” and zero in case of “failure.” The probability of success is \( \pi \).

Returns in the second period of production are independent across projects, which means that there is no aggregate uncertainty in the economy. Both the entrepreneur, his project and the capital invested perish, simultaneously, after the second production period. It may be useful to think of failure as a random event which destroys capital after the first production period.

An entrepreneur can affect the probability of “success,” \( \pi \), through the amount of “effort” he puts into the project. We denote the disutility of effort (evaluated at the second production period, in terms of the numeraire) by \( \psi(\pi) \), and assume

\[ \psi(0) = 0, \quad \psi' > 0, \quad \psi'' > 0, \quad \psi'(0) = 0, \quad \psi'(1) = +\infty. \]  \hspace{1cm} (4)

Hence, increasing the probability of success entails a sacrifice of entrepreneurial utility (at an increasing rate). The last two assumptions are made to guarantee that the entrepreneurs’ problem has an interior solution. The assumption about the third derivative guarantees that the solution is a continuous function of the relevant prices (see below).

Entrepreneurs are born penniless, and have to borrow in order to activate their projects. Note, however, that the \( t - 1 \) born entrepreneur has a deterministic cash flow of \( p_t Y \), in the first production period, which is not affected by the agency problem. This source of “liquidity” plays a crucial role in the analysis below.
We assume that effort is not observable by the consumers. It is therefore impossible to write contracts contingent upon the amount of effort the entrepreneur puts into his project. A certain level of effort can be implemented only by making it incentive compatible with the entrepreneur’s self interest. Crucially, we impose no other constraint on the problem, and allow entrepreneurs and financiers to use any observable information they wish so as to minimize the agency problem.

It is worth mentioning that our story is, in essence, the same as in SW. The crucial assumption is that the \( (t-1) \) born entrepreneur may increase his second-period income (net of the disutility of effort), \( p_{t+1} Y - \psi(\pi) \), by increasing the risk of failure. As we show below, the outcome is the same as in SW: when investment is externally financed, entrepreneurs tend to take an excess risk of failure. We differ from SW in that we split net income into an observable (pecuniary) part and a non-observable (non-pecuniary) part. That is done in order to make effort unobservable \textit{ex post}, so that the contract is resilient to a De Meza and Webb [9] sort of criticism. Also, we have a continuum of failure probabilities rather than two (‘‘risky’’ and ‘‘safe’’ in SW), but that is done, mainly, for analytical convenience.

3. THE CONTRACT PROBLEM

In this section we solve the contract problem. It is convenient to consider the problem of the generation born at \( t-1 \) so that \( t \) is the first production period and \( t+1 \) the second production period.

To establish a benchmark, consider the first-best, full-information problem. In that case, the interests of the entrepreneur and the financier are aligned: to maximize the project’s net present value and to activate it if such value is positive. Hence

\[
\max_{\pi} -1 + \left( \frac{1}{1+r} \right) p_{t} Y + \left( \frac{1}{1+r} \right)^2 \left[ \pi p_{t+1} Y - \psi(\pi) \right]. \tag{5}
\]

The first order condition of this problem is

\[
p_{t+1} Y = \psi'(\pi), \tag{6}
\]

which has an ordinary production-theory interpretation. Effort is an input; to find its optimal amount one should equate the value of its marginal product to its marginal cost.

We plot, in Fig. 1, a rotated (by 90°, counter clock-wise) \( \psi' \) curve with its origin at the point \((p_{t+1} Y, 0)\). For reasons to become clear below, we...
call it the IC curve.\footnote{The shape of the IC curve is determined by the assumptions in (4).} It follows from Eq. (6) that the first-best level of effort is at the intersection of the IC curve with the vertical axis (see Fig. 1). Obviously, when the price of second-period output increases, the value of the marginal product of effort increases as well and the input of effort should be increased. We refer to this as the \textit{profitability} effect.

To check whether activating the project is profitable at all, denote the solution of (5) by the function
\begin{equation}
\psi = \tilde{\Pi}(p_{t+1}), \quad \text{where} \quad \tilde{\Pi} > 0,
\end{equation}
and the value of the project by
\begin{equation}
\pi(p_t, p_{t+1}) = 1 + \left( \frac{1}{1+r} \right) p_t Y
\end{equation}
\begin{equation}
+ \left( \frac{1}{1+r} \right)^2 \{ \tilde{\Pi}(p_{t+1}) p_{t+1} Y - \psi[\tilde{\Pi}(p_{t+1})] \},
\end{equation}
which is increasing in both prices. Then the project is activated if and only if its value is positive. Note that $p_t$ has a pure rent effect on profits (and the activation decision), but it does not interfere with the optimal allocation of effort. The reason is that effort is an input in the production of second-period output; hence, it is not affected by first-period prices.

\footnote{The shape of the IC curve is determined by the assumptions in (4).}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{The contract problem.}
\end{figure}
Now the asymmetric-information case. It is important to recognize that the constraint imposed by the asymmetry of information may not be binding. Suppose that first-period revenue is sufficient to pay back the external financiers, namely $p_t Y \geq (1 + r)$. Then, by the time the effort decision is made, the project is already internally financed. Hence the entrepreneur solves the same problem as in (5) and inserts the first-best level of effort into the project.

Consider next the case where first-period income is not sufficient to pay back the financiers, namely $p_t Y < (1 + r)$. The problem is how to guarantee the required repayment to the financiers with a minimal decrease in the entrepreneur’s welfare. A contract should be designed which is contingent upon all relevant, observable information, i.e., the success or failure of the project in the second production period. A feasible contract is a repayment, $R \in [0, p_{t+1} Y]$ in case of success, and zero in case of failure. Technically, the optimal contract maximizes the entrepreneur’s welfare (9), subject to an incentive compatibility constraint (IC)—Eq. (10)—and participation constraints (PC) for both the financier and the entrepreneur—Eq. (11) and (12), respectively:

$$\max_{\pi, R} \left( \frac{1}{1+r} \right)^2 \left[ \pi (p_{t+1} Y - R) - \psi(\pi) \right]$$

subject to:

$$\pi \in \arg \max \left( \frac{1}{1+r} \right)^2 \left[ \pi (p_{t+1} Y - R) - \psi(\pi) \right],$$

$$\left( \frac{1}{1+r} \right) p_t Y + \left( \frac{1}{1+r} \right)^2 \pi R \geq 1,$$

$$\left( \frac{1}{1+r} \right)^2 \left[ \pi (p_{t+1} Y - R) - \psi(\pi) \right] \geq 0.$$ 

By standard considerations one can show that the constraint in (12) is not binding. It follows that the feasibility set is defined by Eq. (10) and (11) only. First, consider the first-order condition of Eq. (10):

$$p_{t+1} Y - R = \psi'(\pi).$$

Hence, for any repayment $R$, the incentive-compatible level of effort can be found with the aid of the IC curve of Fig. 1: just measure $R$ on the horizontal axis and find $\pi$ on the curve. Next, consider the financier’s participation
constraint (11). The \((R, \pi)\) combinations that satisfy this constraint lie above the PC curve, which is given by

\[
\left( \frac{1}{1+r} \right) p_y Y + \left( \frac{1}{1+r} \right)^2 \pi R = 1. \tag{14}
\]

That is the rectangular hyperbola in Fig. 1. Hence, the feasibility set is defined by the arc of the IC curve between points \(A\) and \(A'\). It is easy to see that moving leftwards, and closer to the first-best point, would increase the value of the objective (9).\(^{16}\) Hence, point \(A\), where (11) holds with equality, is the second-best, optimal contract.

It is obvious that the IC and PC curves may not intersect at all, in which case the feasibility set defined by constraints (10)--(12) is empty. In that case no funds can be obtained by the entrepreneur. Let the boundary of the set of activation prices be given by the \(p_{t+1} = f(p_t)\) function, defined by the tangency of the IC and PC curves. Then, the project is activated if prices are above \(f\), and is not activated if prices fall below \(f\). It is easy to see that \(f\) is downward sloping.

Hence, the optimal contract is characterized by three “regimes:” internal finance, external finance and no activation as follows:

\[
\pi_t = \Pi(p_t, p_{t+1}) = \begin{cases} 
(p_t > (r + 1)/Y) & \text{if } p_t > (r + 1)/Y \\
\text{“point } A\text{”} & \text{if } p_t < (1 + r)/Y \text{ and } p_{t+1} > f(p_t) \\
0 & \text{if } p_{t+1} < f(p_t)
\end{cases}
\] \tag{15}

Let us summarize the solution with the aid of Fig. 2, where the three regimes are clearly visible.

(i) Internal finance: this regime is effective when \(p_t\) is sufficiently high for the project to be financed out of the first-period deterministic income. Higher first-period prices will increase rents but will have no effect on the allocation of effort as it is already at the first-best level. Hence, the \(\Pi\) function is flat with respect to \(p_t\) (see Fig. 2).

(ii) External finance: this regime is effective for interim \(p_t\)'s such that the project cannot be internally financed, but is still activated. It is clear from Fig. 1 that effort is below the first-best level. Further, \(\Pi_t > 0\); as \(p_t\)

\(^{16}\)To prove this claim diagrammatically notice that the area below the IC and right of \(R\) represents the value of the objective multiplied by \((1+r)^2\).
increases, effort increases continuously\(^{17}\) and approaches its first-best level. The reason is straightforward: \(p_t\) is a source of liquidity which allows the entrepreneur to mitigate the distortionary effect of external finance. Hence, the \(\Pi\) function is upward sloping with respect to \(p_t\) (see Fig. 2). The crucial difference between this regime and the one above is in the presence of this liquidity effect: whether first-period income has a pure-rent or an allocational effect. Note also that \(\Pi_t > 0\) (see Fig. 1), thus, since \(\Pi_t' > 0\) as well, the whole curve in Fig. 2 shifts upwards when \(p_{t+1}\) increases.

(iii) No activation: this regime is effective when \(p_t\) is very low. Financial requirements are so high that incentive-compatible effort falls to a level at which financiers cannot get the market return on their funds. Finance is not supplied, the project is not activated, and effort jumps discontinously to zero.\(^{18}\)

\(^{17}\) Within the external finance regime continuity is guaranteed by \(\phi'' > 0\) (see Fig. 1); continuity is preserved at the switch from the external to the internal finance regime: as \(p_t\) approaches \((1+r)/Y\) (from below) the PC curve collapses to the axes and effort approaches the first best level.

\(^{18}\) When the IC and the PC curves are tangent \(\pi\) is still strictly positive.
For the sake of the welfare analysis in Section 6, we define

\[
v(p_t, p_{t+1}) = -1 + \left( \frac{1}{1+r} \right) p_t Y \\
+ \left( \frac{1}{1+r} \right)^2 \{ I(p_t, p_{t+1}) p_{t+1} Y - \psi[ I(p_t, p_{t+1})] \},
\]

which represents the present value of entrepreneurial profits, provided the project is activated at all. It is easy to check that the two partial derivatives of (16) are positive.

4. AGGREGATE SUPPLY AND MARKET EQUILIBRIUM

Credit rationing is a possibility in our model. Intuitively, suppose that prices are \( p_t \) and \( p_{t+1} = f(p_t) \) such that the IC and the PC curves are just tangent. These prices are demand determined and some \( (t-1) \) born entrepreneurs do not participate in the market. Now what would happen if they participated? Prices would fall further below, which would drive all entrepreneurs into the no activation regime. Hence the credit rationing.

We focus, below, on no-rationing equilibria because they are simpler to analyze and sufficient to illustrate the functioning of the reversion mechanism in which we are interested.

Suppose there exist an equilibrium with no rationing at any point on the equilibrium path. (We provide a condition that guarantees the existence of such an equilibrium at the end of this section.) Hence all entrepreneurs participate in the market, and the market-clearing condition in period \( t+1 \) is simply

\[
[1 + I(p_t, p_{t+1})] Y = D(p_{t+1}).
\]

Note that the \( t+1 \) supply is made up of the output of all the \( t \) born entrepreneurs who are producing for the first time, and the successful \( t-1 \) born entrepreneurs who are producing for the second time.

Equation (17) defines a first-order difference equation in prices. Denote it by

\[
p_{t+1} = g(p_t).
\]

\( ^{19} \) Equation (14) is obviously valid for the internal finance regime, but also for the external finance regime. To see the latter, one can use the fact that the financier’s participation constraint (11) is binding to rewrite (9).
The function $g$ has two properties which are essential to our analysis. First, if projects are internally financed, i.e., $p_t \geq (1+r)/Y$, then the market-clearing condition is

$$[1 + \Pi(p_{t+1})] Y = D(p_{t+1}).$$

The solution of (18) in terms of $p_{t+1}$ is unique. Denote it by $\bar{p}$. It follows that, for $p_t \geq (1+r)/Y$, $p_{t+1}$ equals $\bar{p}$ as Fig. 3 shows. Intuitively, projects are internally financed, so that higher period $t$ prices create additional rents, but rents do not affect effort, so the next period price does not change in response.

On the other hand, if $p_t < (1+r)/Y$, $g$ is downward sloping

$$g'(p_t) = -\frac{\Pi_1}{\Pi_2 - D} < 0. \quad (19)$$

Notice that $g$ is continuous at point $(1+r)/Y$ (see Fig. 3 again) due to the corresponding continuity of $\Pi$. As for the magnitude of $g'$ (to the left of $(1+r)/Y$), note that the more responsive demand is to changes in prices, the flatter the curve is. It is useful to describe two limiting cases:

(i) Inelastic demand. As $D' \to 0$, $g$ approaches the level set defined by $\Pi(p_t, p_{t+1}) = \Pi(\bar{p})$. Using (13) and (14) one can verify that this level set is linear with a (downward) slope of $-(1+r)/\pi < -1$. 

![Fig. 3. The difference equation.](image-url)

\[\text{Fig. 3. The difference equation.}\]
(ii) Perfectly elastic demand. If $D' \to -\infty$, $g$ will be flat and equal to $\bar{p}$.

Thus, for intermediate values of $D'$, the $g$ function lies anywhere in the triangle below the above mentioned level set, and the horizontal line $p_{t+1} = \bar{p}$. So, it turns out, the slope of $g$ may be greater than one (in absolute value).

Having discussed the law of motion, let us look at the initial conditions. Suppose that at $t = 0$ there is a measure one continuum of entrepreneurs (born at $t = -1$) who have only one production period left. Given their wealth, they choose an effort level $\pi_0$ such that the initial price, $p_1$, is determined by

$$(1 + \pi_0) Y = D(p_1).$$

Since there exist a one-to-one mapping from the wealth of the initial generation to $p_1$, we consider the initial price $p_1$ as a given data for our economy.

We can now state a sufficient condition for a no-rationing equilibrium. Obviously, any pair of consecutive prices $(p_t, p_{t+1})$ should lie above the graph of the $f$ function. So consider the case where $g$ and $f$ intersect like in Fig. 3; if the point $(\bar{p}, \bar{p})$ lies above $f$, then whenever $p_t$ exceeds $(1 + r)/Y$, the following $p_{t+1}$ and the whole continuation equilibrium sequence will be above $f$; if, in addition, the initial point is above $f$ the equilibrium path will have no rationing at any point. Hence, a sufficient condition for a no-rationing equilibrium is

$$\bar{p} > f(\bar{p}), \quad \text{and} \quad p_1 \geq \bar{p}. \quad (20)$$

That condition (20) can be satisfied at all is clear from the fact that if the demand $D$ became larger the graph of $g$ would shift upwards and to the right, whereas the graph of $f$ would remain unchanged. Hence, for some demand schedules this sufficient condition can be satisfied.

Before we continue, let us just point out that the downward sloping segment of $g$ captures the basic intuition of our model. When the period $t$ price of coffee increases, entrepreneurs are more liquid. They are thus less dependent on external finance, which gives them an incentive to increase $\pi$. That increases the quantity supplied next period and decreases prices. So next period entrepreneurs would be less liquid. Hence, a cobweb sort of dynamics appears and cycles may be generated.

20 Namely: for any initial price $p_1$ there exists a level of initial wealth, $g^{-1}(p_1) Y$, such that $\pi_0$ is a rational choice for a perfectly-foreseen $p_1$. 
5. DYNAMICS, STEADY STATES AND CYCLES

Denote the stationary point of $g$ by $p^*$. Then, given an initial price $p_1$, three types of equilibria can emerge

(i) If the point $[(1 + r)/Y, \tilde{p}]$ lies to the left of the 45° line then $p^* = \tilde{p}$. The system would converge to its stationary point at $t = \tilde{t}$, the latest.

(ii) If the point $[(1 + r)/Y, \tilde{p}]$ lies to the right of the 45° line then $p^* > \tilde{p}$. If $|g'(p^*)| < 1$, the system would converge to $p^*$ with short run oscillations which would die out, eventually.

(iii) If $p^* > \tilde{p}$ (like in the previous case), but the system cannot converge to its stationary point (say) because $|g'(p^*)| > 1$, then the system would converge (after a finite number of periods) to a (two-period) periodic equilibrium as in Fig. 4. The only exception is when the initial price happens to equal $\tilde{p}$. Since the high price is associated with a contraction of supply we call it the “bust price.” By the same logic, we call the other price the “boom price.” Note that entrepreneurs live through both a boom and a bust, but they face different sequences of the two prices depending on whether they start to produce in a boom (“boom start-ups”) or in a bust (“bust start-ups”).

A few points are in place here. The stationary cycle may not be unique. But all stationary cycles have a periodicity of two. Further, the whole equilibrium path is uniquely determined by the $g$ function and the initial condition. If there are many stationary cycles, the initial price will determine to which of these the system will converge. Our story involves no element of multiplicity of equilibria. Note also that the system cannot “jump” to $p^*$ by means of saddle path convergence because of the tight correspondence between initial prices and initial wealth as discussed above.

It is obvious that without the informational problem, the system would quickly converge to the stationary price $\tilde{p}$. That reflects some fundamental differences in the way a moral-hazard economy operates, relative to the full-information one. As already emphasized, rents have no allocational role in the full-information economy. In that case $p_{t+1}$ is not affected at all by $p_t$, and can “freely” jump to $\tilde{p}$. The whole dynamic relation between $p_t$ and $p_{t+1}$ results from the mechanics of the contract and the agency problem. Without that mechanism cycles are not generated. In fact one

\footnotesize
21 At least from a neighborhood of $p^*$.
22 It is well-known that it takes a non-monotonic (first-order) difference equation to produce higher order cycles, see Grandmont [13].
23 As indicated, a non-negative net present value condition should be satisfied. It is easy to see that condition (20) ensures that.
should be more careful about that argument: the discussion above already assumes (via the restriction on the location of \( f \)) that the demand for coffee is high enough so that all entrepreneurs get sufficiently high rents and participate in the market. What would happen in the full information case if demand is not high enough to assure positive net present value under full participation? Can the dynamics of partial entry generate cyclical prices? The answer is no. In Appendix A we explore partial entry dynamics with a binding “zero profit” condition \( \bar{\eta}(p_t, p_{t+1}) = 0 \), and prove that this equilibrium has a unique saddle path convergence to a stationary point. (Note that in this case, unlike in the asymmetric information case entrepreneurs are indifferent between participating in the market and staying out of it.) Hence, a full-information economy will not oscillate even without the restrictions imposed on the location of the demand schedule. This property gives extra power to our claim that moral hazard is the very ingredient that generates cycles in our model.

It is obvious that the economy is more cyclical the steeper is the \( g \) function. Looking again at Eq. (18) we can relate its slope to the liquidity and the profitability effects mentioned above (in relation to Fig. 2). \( g \) is steeper the stronger is the liquidity effect \( \Pi_1 \), and the weaker is the profitability effect \( \Pi_2 \). That a strong liquidity effect contributes to cyclicality is in line with the main thrust of the paper: entrepreneurs depend more on external finance, and effort is reduced further away from the first best. Note, however, that a weak profitability effect contributes to cyclicality. To
understand why, consider an entrepreneur who starts to produce in the bust
(see Fig. 4). Obviously he is little liquid and may be tempted to choose a
low level of effort. But then, he anticipates that other entrepreneurs will do
the same, generating high second period prices which will bring about high
profits. This will push him in the direction of a higher level of effort. The
more effort he puts, the lower are next period prices, the flatter is the $g$
function, and the smaller is the magnitude of output fluctuations.

So flows of wealth in and out of the entrepreneurial sector keep on fueling
the cycle. A boom leads to a bust and the bust to a boom. Importantly,
no constraints on rationality, either via expectations formation or sub-
optimal contracting are imposed. In particularly, we do not assume
indexation away: all contracts written from period one onwards make use
of all available information, including the future price of coffee. But this
sort of indexation does not provide insurance and it does not smooth
entrepreneurs' income. We may think about it in the following way: since
the state of the world (i.e., the initial $p_1$) is already realized when an
entrepreneur is born, insurance markets are closed by a standard Hirshleifer
[15] argument. On the other hand, our theory depends on some initial,
maybe just a little deviation, from the stationary price $p^*$. We build no
theory to explain the initial discrepancy, but we show that the cycle can
persist even as we get arbitrarily further away from period one.24

6. STABILIZATION: WELFARE ANALYSIS

Now to the policy issue: can a stabilizing policy be implemented, and
could such a policy be justified on the basis of some welfare accounting?
We focus on the case in which the economy is at the periodic equilibrium
as in Fig. 4, with alternating “boom” (low) and “bust” (high) prices, $p$ and
$p^*$ respectively (see Fig. 5). Obviously, entrepreneurs who start up produc-
tion in the bust (when prices are high) put the first-best amount of effort
into their projects. Hence, there is no point in trying to improve on them.
But those who start up in the boom (when prices are low) are excessive
risk-takers. Getting them closer to the first-best level of effort would require
enhancing the liquidity or the profitability of their projects by means of a
subsidy. Suppose that is done by an expansionary demand policy in the
bust, just like in an old-fashioned macroeconomics textbook.

Consider a perfectly foreseeable policy that allocates a subsidy $s$ per unit
of output to old boom start-up entrepreneurs, in a bust period.25 Note that

24 Unlike in Kiyotaki and Moore [20] where the cycle dies out, eventually, and it takes an
additional “unanticipated” shock to start it all over again.

25 A policy giving a subsidy per unit of first period output or per investment project to each
generation of boom start-ups has an identical impact.
the policy is discriminating: entrepreneurs who start producing in the bust do not get the subsidy. To see the effect of the subsidy, use the implicit function theorem on the market-clearing condition for the bust price

\[\left[1 + \Pi(\bar{p}, \hat{p} + s)\right] Y - D(\hat{p}) = 0,\]

in order to set

\[-1 < \frac{\partial \hat{p}}{\partial s} = -\frac{\Pi_2 Y}{\Pi_2 Y - D'(\hat{p})} < 0.\] (21)

Consider the boom start-ups and assume, for the moment, that they face the same boom price, \(\bar{p}\), as before. The subsidy drives these entrepreneurs closer to the first-best level of effort. Hence, bust quantities are expanded (see the broken line in Fig. 5); the market-price of coffee—as seen by coffee buyers and bust start-ups (who do not get the subsidy)—falls to \(\hat{p}'.\) Since the price falls by less than the subsidy (see (21)), the price, as seen by the boom start-ups increases to \(\hat{p}' + s.\) Note that if the subsidy is not too big, the lower bust price will not affect the effort of the bust start-ups, who are already at the first best. Hence, the boom price, \(\bar{p}\), is indeed unaffected. It follows that the price combination observed by the boom start-ups is given by point \(B\), while the price combination observed by the bust start-ups is given by point \(C.\) But then, the market (i.e., buyers) price combination is given by point \(A.\) Obviously, the amplitude of boom-bust market prices
is decreased by the policy: from \((p, \hat{p})\) to \((\hat{p}, \hat{p}')\). Since market prices are monotonic in quantities the policy is, indeed, unambiguously stabilizing.

It is obvious that the bust start-ups are hurt by the policy: they face a lower price when starting-up and the same price when old. Obviously, boom start-ups gain by the policy. Also, coffee buyers gain by facing a lower price in the bust. To see whether the benefits exceed the losses, let us add-up both (they are easy to evaluate in terms of the numeraire); the whole computation is done for a bust period

\[
W = (1 + r)^2 \cdot v(\hat{p}, \hat{p} + s) + (1 + r) \cdot v(\hat{p}, \hat{p}) + U(\hat{p}) - s II(\hat{p}, \hat{p} + s) Y.
\]

\(W\) adds up the (properly capitalized) values of the projects of the boom and bust start-ups, the utility of the consumers, and the cost of the subsidy to the taxpayers.

Differentiating \(W\) with respect to \(s\) and evaluating at \(s = 0\), we get

\[
\frac{\partial W}{\partial s} = (1 + r)^2 \cdot \frac{\partial v}{\partial \hat{p}} \left(1 + \frac{\partial \hat{p}}{\partial s}\right) + (1 + r) \cdot \frac{\partial v}{\partial \hat{p}} \frac{\partial \hat{p}}{\partial s} + U'(\hat{p}) \frac{\partial \hat{p}}{\partial s} - II(\hat{p}, \hat{p}) Y.
\]

(See Appendix B for the derivatives in this and the next equation.) For brevity, denote \(\partial \hat{p} / \partial s\) by \(-\lambda\) (recalling that \(0 < \lambda < 1\) by Eq. (21)), \(II(\hat{p}, \hat{p})\) by \(\hat{\pi}\), and the repayment obligation of the boom start-ups by \(\hat{R}\). Then, using Eq. (3), (8), (16), and \(D(\hat{p}) = (1 + \hat{\pi}) Y\), we can write

\[
\frac{\partial W}{\partial s} = \hat{\pi} \left(\frac{Y - \partial \hat{R}}{\partial \hat{p}}\right) (1 - \lambda) - Y \hat{\lambda} + (1 + \hat{\pi}) Y \hat{\lambda} - \hat{\pi} Y = -\hat{\pi} \frac{\partial \hat{R}}{\partial \hat{p}} (1 - \lambda),
\]

where \(\partial \hat{R} / \partial \hat{p}\) can be computed using (13) and (14). Note that if \(\hat{R}\) were zero (as it is for the bust start-ups), the above derivative would equal zero; this confirms our intuition that there is no room for a subsidy like \(s\) in a boom. If \(\hat{R}\) is positive (as it is for the boom born generation), then \(\partial \hat{R} / \partial \hat{p} < 0\) and the aggregate welfare measure can be improved by choosing a positive \(s\). Intuitively, unlike other agents, the marginal value of income for the entrepreneurs who start to produce in a boom is higher than one, since, in addition to the direct distributive effect, increasing their income has an allocational effect, that pushes them closer to the first best. Thus, when they obtain additional rents by means of the subsidy, they generate added value in excess of the taxpayers’ loss.

From this result, it is tempting to say that Pareto improvements could be achieved by compensating the losers by lump-sum taxation. But this is not true. In a world where rents have a role in providing incentives, lump-sum taxes are not neutral. Consumers and bust start-ups could be lump-sum
taxed without affecting their marginal decisions.\textsuperscript{26} But if the boom start-up entrepreneurs were lump-sum taxed for compensation purposes (say, out of their first-period revenue) that would undo the allocational effect of the subsidy.\textsuperscript{27}

Hence, the question of whether cycles as those described in this model should be stabilized has no clear answer. Stabilization policies are desirable according to a policy criterion which is weaker than Pareto optimality.

7. CONCLUDING REMARKS

As noted above, our modeling gives priority to analytical transparency rather than to realism. Since we model a pure reversion mechanism the system (while in a stationary cycle) reverts each period to its previous state: from boom to bust and vice versa, one period after another. Of course, this is an incomplete description of the fluctuations observed in the real world, as captured by macroeconomic time series.

There is no reason why the reversion and the propagation mechanisms cannot be combined, within a single model, in order to generate equilibria with realistic time-series properties. Namely, after a reversion period the propagation mechanism takes over for several periods and the system grows out of the recession. At a certain point it reverts back to a bust and it all starts over again. It is unlikely, however, that such a combined model can preserve the simplicity and transparency that characterizes the current one. Our experience with related models (work in progress) shows that important properties may vanish: the equilibrium path may not be unique, the dynamic system may be of a higher order, numerical procedures may have to replace analytical arguments. Also, getting a more realistic financial structure may require to depart from complete contracts, thus undermining one of the main points we make. We have thus chosen, in this paper, to put down the bare bones of the theoretical skeleton and prove some basic results. Hopefully, that would provide better foundations for future, more empirically motivated, work.

Meanwhile, it is useful to point out that even in its crude form our model can provide some insight into an important policy question. It is sometimes argued that insufficient indexation, especially in the banking sector is the source of financial instability. The remedy would be to make banks' assets

\textsuperscript{26} It is immediate to check that the consumers' gain from the subsidy does not suffice to compensate for both its direct cost to the taxpayers and the losses caused to the bust start-ups.

\textsuperscript{27} Strictly speaking, entrepreneurs cannot be lump sum taxed in the second period. Since they have zero wealth in case of failure, any tax is necessarily state contingent, and can be avoided in probability by excessive risk taking. Hence, it is not neutral.
and liabilities more responsive to market conditions, either through indexation or via securitization. It is argued that by the mere replacement of banks by mutual funds, much of the problem can be resolved.\footnote{See Mankiw [21] for an argument along these lines, though more in the context of liquidity provision and bank-runs.} Our analysis does not support that sort of an argument. Rather, it shows that even after contracts are made responsive to any relevant market price, “financial instability” is still a basic fact of life; an inherent feature of a competitive market economy subject to moral hazard.

**APPENDIX A**

*Existence and Uniqueness of a Saddle-Path in the Full-Information Economy*

This Appendix analyzes first-best full-information dynamics. We prove that the system converges, within at most one period, to an equilibrium with stationary prices and stationary total output. It has been shown in the text that the optimal choice of \( \pi \) by the entrepreneurs facing prices \((p_t, p_{t+1})\) is \( \bar{\pi}(p_t, p_{t+1}) \) and the associated net present value of the project is \( \bar{\pi}(p_t, p_{t+1}) \). Investment is feasible if and only if \( \bar{\pi}(p_t, p_{t+1}) \geq 0 \). In fact, when \( \bar{\pi}(p_t, p_{t+1}) = 0 \) entrepreneurs are indifferent between investing and not investing, and we may have a situation where only a fraction \( q_t < 1 \) of the entrepreneurs who can start up production at \( t \) activate their projects at \( t-1 \). Allowing for that possibility, the dynamics of the system can be described by the market-clearing condition

\[
[q_{t+1} + q_t \bar{\pi}(p_t, p_{t+1})] Y = D(p_{t+1}),
\]

and the free-entry condition

\[
\begin{cases}
\bar{\pi}(p_t, p_{t+1}) > 0 & \text{and } q_t = 1 \\
\bar{\pi}(p_t, p_{t+1}) = 0 & \text{and } q_t \in [0, 1] \\
\bar{\pi}(p_t, p_{t+1}) < 0 & \text{and } q_t = 0.
\end{cases}
\] (A2)

Notice that \( \bar{\pi}(p_t, p_{t+1}) = 0 \) defines a first-order difference equation in \( p_t \) with

\[
dp_{t+1}/dp_t = (1 + r)/\bar{\pi}(p_t, p_{t+1}) < -1.
\] (A3)

It intersects with the horizontal axis at \( p_t = (1 + r)/Y \). Denote the stationary point of this difference equation by \( \bar{p} \) and recall that \( \bar{p} \) was defined by
Clearly, if $\bar{p} \geq \hat{p}$ then $\hat{p}$ is the stationary equilibrium with $q_t = 1$ for all $t$ and, as shown in the text, the system converges to this steady state immediately.\footnote{Actually, the demonstration in the text is for an initial value of $q_0 = 1$. But it extends immediately to other initial values, only that convergence will then take one period.} If $\bar{p} < \hat{p}$ the stationary price is $\hat{p}$ and the stationary $q_t$ is the value $\hat{q}$ which satisfies $[1 + \bar{H}(\hat{p})] \hat{q} = D(\hat{p})$. We now show that convergence to the stationary price $\hat{p}$ (within, at most, one period) is the unique equilibrium in the system. The proof is done in two steps.

**Step 1: Existence of an equilibrium convergent to $\hat{p}$.** Let $q_0$ and $\pi_0$ represent the initial conditions of the system in terms of the fraction and effort decision of the entrepreneurs who start up production at $t = 0$ and whose projects are activated. Consider the following two cases:

(i) $q_0, \pi_0 \in [0, [1 + \bar{H}(\hat{p})] \hat{q}]$. Then $p_t = \hat{p}$ and $q_t = [1 + \bar{H}(\hat{p})] \hat{q} - \bar{H}(\hat{p}) q_{t-1}$ for $t = 1, 2, \ldots$ is an equilibrium of the system given by (A1) and (A2).

(ii) $q_0, \pi_0 \in ([1 + \bar{H}(\hat{p})] \hat{q}, 1]$. Then $p_1 < \hat{p}$ and $p_t = \hat{p}$ for $t = 2, 3, \ldots$, and $q_1 = 0$ and $q_t = [1 + \bar{H}(\hat{p})] \hat{q} - \bar{H}(\hat{p}) q_{t-1}$ for $t = 2, 3, \ldots$, where $p_1$ solves $q_0, \pi_0 Y = D(p_1)$, is an equilibrium of the system given by (A1) and (A2).

Checking the previous assertions is immediate. Notice that in (ii) $\bar{v}(p_t, p_{t+1}) < \bar{v}(\hat{p}, \hat{p}) = 0$ since $p_1 < \hat{p}$. Note also that in both (i) and (ii) the convergence of $q_t$ to $\hat{q}$ is asymptotic and non-monotonic. Note also that no agent cares about these oscillations in $q_t$, least of all entrepreneurs themselves who are on the zero profit condition (A1) along the whole process.

**Step 2: Uniqueness.** We now show that there are no oscillations in prices after $t = 2$. Suppose that there exists an equilibrium with such oscillations. Notice, first, that we can rule out the possibility of having oscillations with $\bar{v}(p_t, p_{t+1}) > 0$ for all $t$ since that dynamics is explosive according to (A3); two consecutive $\bar{v} > 0$ or $\bar{v} < 0$ periods can be also excluded. Thus, oscillations must involve either (a) periods of $\bar{v} > 0$ between periods of $\bar{v} < 0$ or (b) periods of $\bar{v} < 0$ between periods of $\bar{v} > 0$. Further, some of these switching periods must dampen the otherwise explosive oscillations.

Suppose (a): the cycle is dampened at $t = s$ with $\bar{v}(p_s, p_{s+1}) > 0$, hence $q_s = 1$. Then, $p_{s-1} < p_{s+1} < p_s$. Using (A1), $D(p_{s+1}) > D(p_s)$ implies

$$q_{s+1} - q_{s-1} \bar{H}(p_s) > 1 - \bar{H}(p_{s+1}).$$

\footnote{Actually, the demonstration in the text is for an initial value of $q_0 = 1$. But it extends immediately to other initial values, only that convergence will then take one period.}
Similarly, as $D(p_{s-1}) \geq D(p_{s+1})$ and $\bar{H}$ is increasing, $q_{s-1} - q_{s+1} \geq \bar{H}(p_{s+1}) - q_{s-1} \bar{H}(p_{s-1}) > 0$. Then,

$$q_{s+1} - q_{s-1} \bar{H}(p_{s}) \leq q_{s+1} - q_{s-1} \bar{H}(p_{s}).$$  \hfill (A5)

Using the property of $\bar{H}$ again, we get

$$q_{s+1}[1 - \bar{H}(p_{s})] < 1 - \bar{H}(p_{s+1}).$$  \hfill (A6)

Combining (A5) and (A6) we contradict (A4). A similar contradiction can be obtained for (b) where $\bar{v}(p_{s}, p_{s+1}) < 0$ and $p_{s} < p_{s+1} < p_{s-1}$.

From here we deduce that the saddle-path convergence to the stationary price described in Step 1 is the unique equilibrium of the system given by (A1) and (A2).

**APPENDIX B**

*Derivatives Used in the Welfare Analysis*

The partial derivatives of the solution of the contract problem for the external finance regime are

\[
\frac{\partial \pi}{\partial p_{t}} = \frac{(1 + r) Y}{\pi \psi^{\prime}(\pi) - R},
\]

\[
\frac{\partial \pi}{\partial p_{t+1}} = \frac{\pi Y}{\pi \psi^{\prime}(\pi) - R},
\]

\[
\frac{\partial R}{\partial p_{t}} = \frac{\psi^{\prime}(\pi)(1 + r) Y}{\pi \psi^{\prime}(\pi) - R},
\]

\[
\frac{\partial R}{\partial p_{t+1}} = \frac{R Y}{\pi \psi^{\prime}(\pi) - R}.
\]

Notice that $\pi \psi^{\prime}(\pi) - R > 0$ because of the relative slopes of the IC and PC curves at the optimum. The partial derivatives of Eq. (16) for the external finance regime are

\[
\frac{\partial v}{\partial p_{t}} = \frac{H(p_{t}, p_{t+1})}{1 + r} \frac{\partial R}{\partial p_{t}},
\]

\[
\frac{\partial v}{\partial v_{t+1}} = \frac{H(p_{t}, p_{t+1})}{1 + r} \left( Y - \frac{\partial R}{\partial p_{t+1}} \right).
\]
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